

A Unified Framework for Computer-Aided Noise Analysis of Linear and Nonlinear Microwave Circuits

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Abstract—This paper introduces a new unified theoretical concept for noise analysis in analog and microwave circuits. Based on the adjoint system approach an analysis technique for general purpose CAD-applications is presented. The algorithm is easy to be implemented into existing CAD-tools. Moreover, it exploits the advantages of sparse matrix techniques.

I. INTRODUCTION

NOISE analysis of analog circuits has been a major research topic for the last 50 years. The theories developed up to now mainly deal with the analysis of twoport circuits based on correlation matrices to achieve a complete characterization of the noise behavior of the circuit. Basically, the analysis is carried out by rebuilding the circuit by twoport interconnections, where the signal transfer and correlation matrices are calculated step by step [1]. This technique tends to get very complicated if the circuit is large. It seems to be nearly impossible to handle e.g., a complete operational amplifier in this way. In the early seventies a new technique for large circuits with uncorrelated noise sources was introduced in [2], which is restricted to the computation of noise voltages. However, considering e.g., a microwave amplifier, this algorithm cannot give a complete description of the noise behavior in terms of the four twoport noise parameters. In [3] the theory of correlation matrices for twoport networks is expanded to n -port networks. This generalization leads to a nodal based noise analysis, which can handle arbitrary linear circuits. However, in an implementation of this method the inversion of the complete nodal admittance matrix should be avoided, by the reduction of network equations [4] using sparse matrix methods. Another approach for general purpose noise analysis is based on noise power waves [5] and specially in combination with the connection scattering matrix [6], [7]. The basic ideas of our work are also applicable in the latter approach, which eliminates the need of the inversion of the large connection scattering matrix. But this will be a part of future work.

Manuscript received March 29, 1991; revised August 8, 1991.

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IEEE Log Number 9103933.

In the area of nonlinear circuits, noise analysis has been restricted to a few specialized circuits up to now. Only in [8] an algorithm based on a piecewise harmonic balance technique and the noise description by n -port correlation matrices is discussed. In the authors opinion the implementation of the circuit partitioning concept used in the piecewise harmonic balance technique is more complicated than the implementation of the nodal harmonic balance. Nevertheless, both methods have their advantages and restrictions, which are discussed in [9]. In general the choice between these two methods will depend on the ratio between the number of nodes connected to nonlinear elements and the total number. The piecewise HB is preferable for a small ratio and the nodal HB for a large ratio. Therefore the application of both methods to nonlinear microwave circuits has been reported in the literature, e.g., [10], [11]. Especially the nodal harmonic balance is in the scope of this paper, because no general noise analysis algorithm which is compatible to this technique has been presented up to today.

The work described here combines the advantages of the techniques used in [1], [3], [2], [8] to a unified theoretical framework. This yields an algorithm, which can handle circuits of arbitrary topologies. Moreover, it can easily be implemented into existing simulators, which utilize the modified nodal analysis, e.g., the ac-analysis in SPICE, mainly because only a modified forward and backward substitution procedure is required plus a description of the noise current sources of the devices.

II. THE LINEAR CASE

For the sake of generality, the algorithm will be described for circuits which are subject to a modified nodal analysis. Moreover, the technique will be discussed for linear circuits first. This means no restriction because the application to nonlinear circuits is straightforward.

The aim of noise analysis is to replace the circuit's internal noise sources, which may or may not be correlated, by a set of equivalent correlated noise sources at the ports of the circuit. In Fig. 1 the commonly used noise equivalent circuits for the twoport case are given, together with the corresponding correlation matrix descrip-

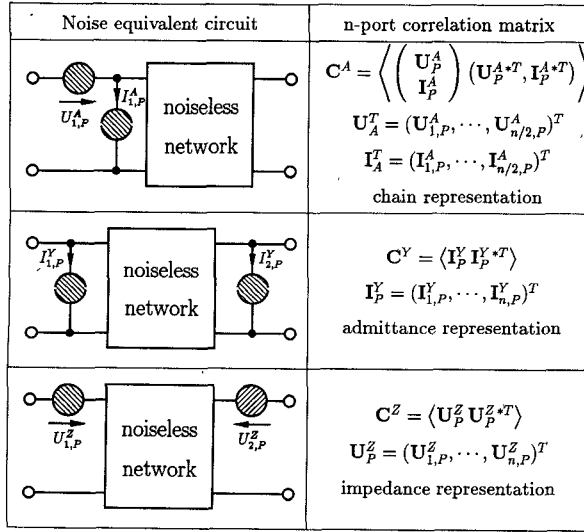


Fig. 1. Noise equivalent twoport circuits and correlation matrix description for the n -port case.

tion for the n -port case. The equivalent noise voltages or currents for the case of an impedance or admittance representation can be measured in a simulation as the open circuit noise voltage or the short circuit noise current, respectively, due to the internal noise sources of the network at the ports, as shown in Fig. 2.

In both cases all quantities are derived from a single analysis; therefore it is possible to keep track of the correlation of internal sources, as it will be shown later. The measurement of the chain representation of the noise equivalent sources would require two different load conditions at the ports, which requires two independent circuit analyses. This results in the problem that the correlation of the noise sources cannot be accounted for. Due to this only the admittance and impedance representation of the noise equivalent sources are subject to the nodal based noise analysis technique presented in this paper. This does not mean that only one kind of internal noise sources, either current or voltage sources, are considered.

The problem of noise analysis of a network can be treated in a more general way by considering the circuit as a general linear system. The currents and/or voltages of the internal noise sources are considered as input variables, where the voltages or the currents of the equivalent port noise sources are considered as output variables. From this point of view the problem is just the calculation of a set of transfer functions from the input towards the output, as indicated in Fig. 3. The approach presented in this paper utilizes the adjoint system method for the calculation of the required transfer functions. The adjoint system method was introduced by [2] in the early seventies concentrating on an adjoint network interpretation. Therefore the mathematical representation of the adjoint system has been interpreted as a network, which reduces the flexibility of its use. More recent publications

[12], [13] lead to the adjoint system method, which is basically a mathematical method.

For the explanation of our nodal noise analysis technique we consider first a network with only one internal noisy device. The noise behavior of this device may be characterized by a set of m correlated noise current and voltage sources, which yields

$$W^d = (I_1, I_2, \dots, I_l, U_{l+1}, \dots, U_m)^T \quad (1)$$

the vector of the device's noise sources and

$$C^d = \langle W^d W^{d*T} \rangle \quad (2)$$

the correlation matrix of these sources. From the system point of view, the vector W^d represents the input variables W_i as shown in Fig. 3. The set of output variables are the currents or voltages of the equivalent port noise sources, which yields depending on the required representation

$$X^n = \begin{cases} (U_1^Z, U_2^Z, \dots, U_n^Z)^T & \text{impedance representation} \\ (I_1^Y, I_2^Y, \dots, I_n^Y)^T & \text{admittance representation} \end{cases} \quad (3)$$

the vector of equivalent port noise current or voltage sources, respectively. Defining the matrix of the transfer functions

$$T^n = [T_{ij}^n] \quad i = 1, n \quad j = 1, m, \quad (4)$$

the relation between the internal and the equivalent noise sources is given by

$$X^n = T^n W^d. \quad (5)$$

Under the assumption that the correlation matrix of device's noise sources (2) is known from an appropriate device model, as well as the matrix of transfer functions, the correlation matrix of the equivalent port noise sources can be calculated as

$$\begin{aligned} C^n &= \langle X^n X^{n*T} \rangle \\ &= \langle (T^n W^d) (T^n W^d)^* T \rangle \\ &= \langle (T^n W^d) (W^{d*T} T^{n*T}) \rangle \\ &= T^n \langle W^d W^{d*T} \rangle T^{n*T} \\ &= T^n C^d T^{n*T}. \end{aligned} \quad (6)$$

The correlation matrix C^n calculated from (6) may be an impedance or admittance representation, depending on the required equivalent circuit. The remaining problem of the nodal noise analysis is the calculation of the elements of the transfer function matrix, which can be solved by treating the transfer functions as sensitivities. Therefore the solution of this problem is the adjoint system method.

Considering a linear circuit, the modified nodal equations are given by

$$\begin{bmatrix} Y_n & A^T \\ Y_b A^T & Z_b \end{bmatrix} \begin{bmatrix} U_n \\ I_b \end{bmatrix} = \begin{bmatrix} I_q \\ U_q \end{bmatrix}, \quad (7)$$

which has the form

$$TX = W. \quad (8)$$

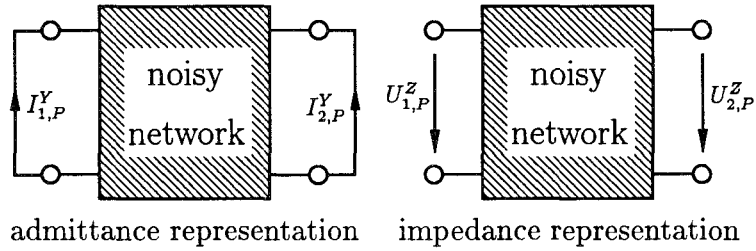


Fig. 2. Circuit configuration for measurement of the values of noise equivalent sources.

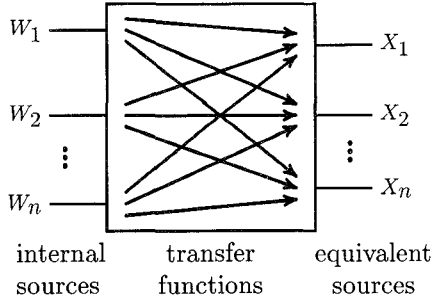


Fig. 3. System interpretation of noise analysis.

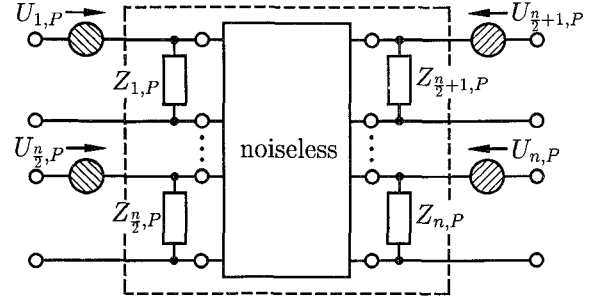


Fig. 4. Noise equivalent circuit including loads.

Under the assumption, that the required output variables X_i^n (3) can be expressed as a linear combination of the elements i' and i'' of the solution of (8), i.e.,

$$X_i^n = (e_{i'} - e_{i''})X = d_i^T X, \quad (9)$$

the resulting adjoint system

$$T^T \hat{X}_i = d_i, \quad (10)$$

can be solved for the adjoint vector \hat{X}_i . The contribution of an internal source W_j to the equivalent noise source at port i is then given by

$$X_{ij} = T_{ij}^n W_j, \quad (11)$$

where

$$T_{ij}^n = \hat{X}_i^T (e_{j'} - e_{j''}) \quad (12)$$

is the transfer function from the internal source W_j to the output X_{ij} . The port correlation matrix is calculated using (6).

In the above explanation only one noisy device with any number of correlated noise current or voltage sources has been considered. If there are two or more noisy devices, which are normally not correlated, the port correlation is given by

$$C_{\text{port}} = \sum_{\text{devices}} C_i^n, \quad (13)$$

where C_i^n denotes the contribution of a single device calculated by (6). If there is a correlation between the noise behavior of two devices, they should be treated as a single device.

In a simulation software the outlined algorithm will be integrated in the circuit simulation. Therefore the LU-factorized MNA matrix T is available from the actual signal analysis. Due to this, the algorithm requires only

one additional modified forward and backward substitution for each port or output variable, respectively to solve the adjoint system (10). Moreover this is the reason why the calculation of the impedance representation of the noise correlation matrix is preferred [14]. The admittance representation would require short circuits at the external ports, which results in a modified circuit and therefore in a completely new analysis.

Typically the internal analysis of CAD-programs will terminate the ports with the ports own reference resistance to calculate the signal behavior, e.g., scattering-parameters. This causes some problems in the noise analysis. To overcome this problem, the n -port is augmented by the reference impedances and the noise analysis carried out using the equivalent circuit in Fig. 4.

The resulting impedance representation of the n -port's noise correlation matrix is transformed into the corresponding admittance representation by using the n -port admittance matrix of the circuit in the dashed box as transform matrix according to [3]. If the loads are considered as noise free this admittance representation describes the noise behavior directly. Otherwise it has to be corrected by subtracting the contribution of the loads. All required results of the noise analysis e.g., the noise or the minimum noise figure can be calculated from this admittance representation of the noise correlation matrix. A detailed and general discussion of n -port noise characterization by correlation matrices has been given by Russer in 1976 [3] and 1990 [4], respectively. This topic is also discussed in [15].

For verification and comparison purposes, the described algorithm and the nodal based technique given in [3] were implemented, using standard algorithms for matrix inversion and factorization.

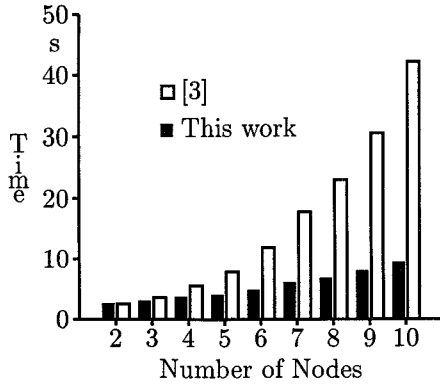


Fig. 5. Comparison of computation times.

In Fig. 5 the computation times for test circuits, with two ports and n nodes are given. The circuits are MES-FET equivalent circuits starting from a very simple one with two nodes and finishing with a complete one with ten nodes for microwave applications. The comparison of the computation times depends strongly on the implementation and the used computer. However, our algorithm exhibits the better performance and its gain in computation time grows with the number of nodes. As a result of the MMIC-technology, the complexity of microwave circuits grows and new CAD-techniques are needed, which can handle large circuits. The presented algorithm combined with an appropriate sparse matrix method may be able to handle circuits with even hundreds of nodes.

III. THE NONLINEAR CASE

In this section the generalization of adjoint nodal noise analysis (ANNA) for nonlinear circuits under large signal-steady state operation is discussed. It will be shown that the ANNA-method is applicable for nonlinear circuits in combination with the harmonic balance method, as mentioned before.

Due to its nature noise is a small signal, which does not disturb the large signal steady state of the circuit. This is the base for noise analysis in nonlinear circuits as commonly used in [16], [17]. Typically a special mixer configuration is treated by considering a dominant LO-signal, resulting in a time-varying equivalent circuit, which is used to calculate the small signal mixing products as well as the noise behavior. This means a linearized description of the nonlinear circuit's transfer properties is used.

Today the analysis of nonlinear circuits in the frequency domain is commonly done using the harmonic balance method [10] in combination with the Newton-Raphson algorithm using an exact analytical formulation of the Jacobian matrix [11]. To outline our noise analysis we use a complex nodal formulation of the HB-equations for the sake of simplicity

$$F(U) = I(U) + j\Omega Q(U) + YU + I_q. \quad (14)$$

Equation (14) describes the harmonic balance error in the frequency domain. For detailed discussion of this formulation refer to [17]. Nevertheless an equivalent real formulation or modified nodal formulation may be used to reduce the computational efforts or to include eg. independent and controlled voltage sources, respectively. If a solution of the nonlinear equation system is found using the Newton-Raphson method, the state vector U_0 yields

$$F(U)|_{U=U_0} = 0. \quad (15)$$

Assuming in this state an additional excitation of the circuit by some kind of small-signal current sources (either noise or signal sources), represented by the vector I_{qn} , results in a new state vector

$$U = U_0 + U_n, \quad (16)$$

as a solution of

$$F(U)|_{U=U_0+U_n} + I_{qn} = 0. \quad (17)$$

Expanding the nonlinear function F into a Taylor series around the state U_0 gives

$$\underbrace{F(U)|_{U=U_0}}_{=0} + \left. \frac{\partial F(U)}{\partial U} \right|_{U=U_0} U_n + I_{qn} = 0, \quad (18)$$

resulting in

$$\left. \frac{\partial F(U)}{\partial U} \right|_{U=U_0} U_n = J(U)|_{U=U_0} U_n = -I_{qn}. \quad (19)$$

The above equations describe the transfer properties of a small-signal in the nonlinear circuit under large signal steady state operation; it exhibits the same structure as (8) and is also a system of linear equations. The basic difference is the fact that it describes the properties of the circuit for the set of frequencies considered in the HB-analysis and not only for a single frequency. Moreover (19) shows that the Jacobian of the final step of harmonic-Newton iteration may be used in the ANNA, depending on the actual implementation. This means the difference between the state for which the Jacobian has been assembled and factorized and the final state must be sufficiently small. The algorithm is the same as in the linear case. It should be noted that the noise sources of active devices are modulated due to the large signal steady state of the circuit. This means that the noise components of the different frequencies are correlated. Therefore a basic problem is the calculation of the correlation between the device's noise sources at the different frequencies. A lot of work in this area has been done for diodes in resistive mixers [16], [18]. For the GaAs MES-FET a first approach is given in [8], introducing modulation functions for the internal noise sources. On the base of [8], the modulated noise sources of the FET can be expressed as

$$\begin{pmatrix} i_g(t) \\ i_d(t) \end{pmatrix} = \begin{pmatrix} h_g(t) & 0 \\ 0 & h_d(t) \end{pmatrix} \begin{pmatrix} i_{g\,dc}(t, U_{dc}) \\ i_{d\,dc}(t, U_{dc}) \end{pmatrix}, \quad (20)$$

where $i_{g\,dc}(t, U_{dc})$ and $i_{d\,dc}(t, U_{dc})$ are the noise currents for the dc components U_{dc} of the steady state U_0 . The

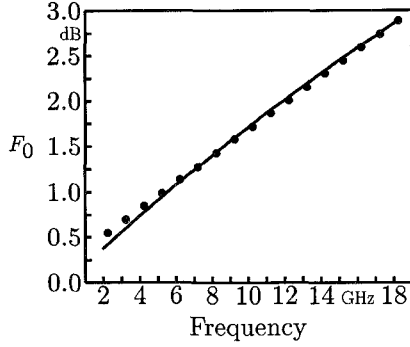


Fig. 6. Minimum noise figure of an inhouse MESFET, \cdots measured, $—$ computed from extracted noise parameters.

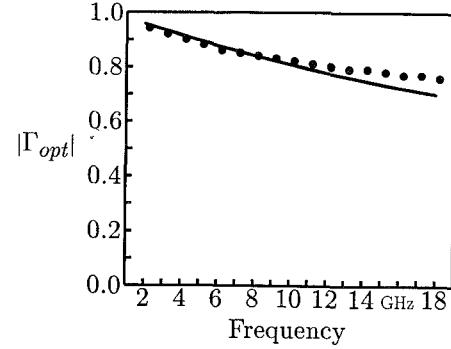


Fig. 7. Magnitude of Γ_{opt} of an inhouse MESFET, \cdots measured, $—$ computed from extracted noise parameters.

correlation matrix between these sources is given by

$$\mathbf{C}^Y(\omega) = 4KT\Delta f \begin{bmatrix} \frac{\omega^2 C_g^2}{g_m} R & -j\omega C_g \sqrt{PR} C \\ j\omega C_g \sqrt{PR} C & g_m P \end{bmatrix} \quad (21)$$

and the modulation functions are

$$h_g(t) = \frac{C_g(u(t))}{C_g(U_{dc})} \sqrt{\frac{g_m(U_{dc})}{g_m(u(t))} \frac{R(u(t))}{R(U_{dc})}} \quad (22)$$

$$h_d(t) = \sqrt{\frac{g_m(U_{dc})}{g_m(u(t))} \frac{P(u(t))}{P(U_{dc})}}, \quad (23)$$

where a detailed explanation of the parameters used is given in [8]. The transformation of (20) into the frequency domain using the conversion matrix formulation yields

$$\mathbf{I}^d = \mathbf{H} \mathbf{I}_{dc}(U_{dc}). \quad (24)$$

In the linear case the noise behavior of a FET is described by two correlated sources, transformed in the nonlinear case into $2H$ correlated sources according to (24). The correlation matrix \mathbf{C}_{dc}^Y of the vector of unmodulated sources $\mathbf{I}_{dc}(U_{dc})$ has only diagonal submatrices, holding the components of (21) at the frequencies of interest. The correlation matrix \mathbf{C}^d of the modulated sources is given similar to (6) by

$$\mathbf{C}^d = \mathbf{H} \mathbf{C}_{dc}^Y \mathbf{H}^{*T}. \quad (25)$$

The noisy nonlinear FET model requires the quantities R , P , and C to be only a function of the time domain steady state $u(t)$. Especially these quantities must not be a function of frequency. A first step in verification of this assumption is the extraction of R , P , and C from small-signal scattering parameters and noise characterization of a GaAs-MESFET. For this purpose the ANNA was utilized to calculate the correlation matrix of FET's gate and drain noise source from the known correlation matrix of the equivalent port noise sources. The results shows a little frequency dependence, which was eliminated by averaging over the frequency range. In the Figs. 6–9 a comparison between the measured and the computed noise parameter of an inhouse MESFET are given. It is

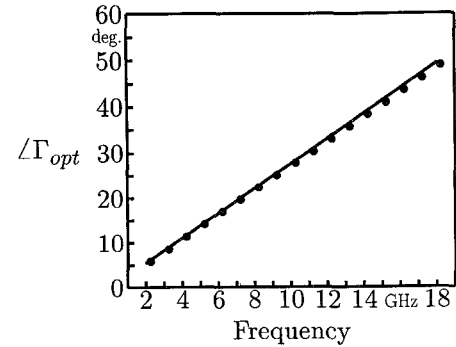


Fig. 8. Angle of Γ_{opt} of an inhouse MESFET, \cdots measured, $—$ computed from extracted noise parameters.

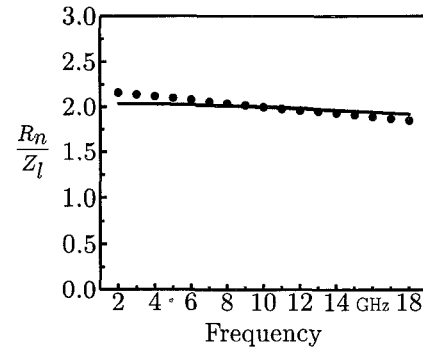


Fig. 9. Equivalent noise resistance R_n of an inhouse MESFET, \cdots measured, $—$ computed from extracted noise parameters.

worth noting that the deviation of the computed results is within the measurement accuracy. Therefore it seems to be reasonable for linear CAD-application to characterize the noise behavior of a MESFET only by the three quantities R , P , and C . Moreover the assumption used in the noisy FET model may be justified.

Finally it should be noted that there are some requirements concerning the accuracy of the Jacobian. In the harmonic-Newton algorithm an inexact Jacobian normally has no direct influence on the accuracy of the nonlinear system's solution. In the ANNA-algorithm errors in the Jacobian are a source of inaccuracy, especially in the analysis of mixer circuits, where a multitone harmonic

balance is required. In this case a Jacobian calculated using the almost-periodic Fourier-transform (APFT) [19] method does not exhibit the required accuracy, due to the limited dynamic range of the APFT method [20]. This potential source of error may be avoided by assembling the Jacobian from the Fourier-coefficients calculated using a multidimensional FFT as shown in [10].

IV. CONCLUSION

An algorithm for the nodal based noise analysis of linear and nonlinear analog circuits has been presented. The method eliminates the need of high dimensional matrix inversions, which occurs in conventional noise analysis algorithms in the case of large scale circuits or nonlinear circuits. Moreover, there are a lot of potential applications for the adjoint nodal noise analysis in the area of noise parameter extraction for a noisy nonlinear FET model [10], where the noise parameters R , P , and C of the intrinsic FET are needed. In this paper we cannot touch all the properties of the presented noise analysis technique, but the method is not limited to the presented applications. In future the ANNA may be utilized in general purpose CAD-tools for nonlinear circuits, covering autonomous as well as autonomous circuits, due to its flexibility, computational efficiency, ease of implementation and the compatibility to existing nodal based harmonic balance methods. In the authors' opinion the noise analysis algorithm based on the adjoint system concept will be a key technique in the development of nonlinear noisy models of MESFET's, HEMT's, and HBT's, because today there is only very little known about these kind of models.

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Ingo Wolff (M'75–SM'85–F'88), for a photograph and biography, see this issue, p. 2068.